Indian Statistical Institute

Back-paper exam. Session 2024-2025

Analysis of Several Variables, B.Math Second Year

Time : 3 Hours   23 December 2024   Maximum Marks : 100   Instructor : Jaydeb Sarkar

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Q1. (20 marks) Is it possible to solve $u = xy$ and $v = x - y$ for x and y as functions of u and v near $(1, 1)$ and $(-1, 1)$? Justify your answer.

Q2. (20 marks) Define $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ by

$$f(x,y) = \begin{cases} \frac{x^2y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Indicate whether the following statement is right or wrong, and justify your answer: f is continuously differentiable.

Q3. (20 marks) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a C^2-function, and let $x_0 \in \mathbb{R}^n$. If $\nabla f(x_0) = 0$ and the Hessian matrix of f at x_0 is positive definite, then prove that there exist $\varepsilon > 0$ and $\delta > 0$ such that

$$f(x_0 + h) - f(x_0) \geq \varepsilon \|h\|^2,$$

for all $\|h\| < \delta$.

Q4. (20 marks) Prove Stokes' theorem for the special case where the surface is the graph of a function $z = f(x, y)$.

Q5. (20 marks) Find the flux of $2xi + y^2 j + z^2 k$ out of the unit sphere.